

Handout 2. Rational and radical expressions, rational and radical equations, inequalities, domain, absolute values

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The purpose of these problems is to review the material covered in the first two midterms.

1. Simplify (by factoring out first) as much as possible:

$$(a) \frac{5x^2 - 8x + 3}{25x^2 - 9}$$

$$(b) \frac{x^2 - 7x + 12}{x^2 - 9}$$

$$(c) \frac{2}{x-1} + \frac{3}{x+1} - \frac{4x-2}{x^2-1}$$

$$(d) \frac{(x^2+1)^2 3x^2 - x^3(2x)(x^2+1)^2}{(x^2+1)^2}$$

$$(e) \frac{1}{(x+1)(x+2)} - \frac{3}{(x-1)(x+2)} + \frac{3}{(x-1)(x+1)}$$

$$(f) (x^2 - 3x + 2) \frac{x^2 - 5x + 4}{x^3 - 6x^2 + 8x}$$

2. Simplify as much as possible (by writing the simplest radical form):

$$(a) \sqrt[3]{\frac{2x^4}{9yz^2}} \quad (b) \frac{\sqrt{x}}{\sqrt{x-1}} \quad (c) \sqrt{x-2} + \frac{2}{\sqrt{x-2}} \quad (d) \frac{\sqrt{x}+2}{\sqrt{x-1}}$$

3. Solve the following equations:

$$(a) \frac{x+5}{x-3} = 7 \quad (b) \frac{6}{x+1} = 5 - \frac{6x}{x+1} \quad (c) \frac{2}{x} + \frac{3}{x+1} = 4$$

$$(d) \sqrt{x+2} = x-4 \quad (e) \sqrt{2x} = \sqrt{x+1} + 1 \quad (f) x^{2/3} - x^{1/3} - 6 = 0 \text{ (substitute } u = x^{1/3}\text{)}$$

4. Solve the following inequalities, using the interval notation in your answers:

$$(a) \frac{2x-3}{3} - \frac{5x+4}{6} > 5 - \frac{3x}{8} \quad (b) 8 < 2x - 7 \leq 5 \quad (c) 0 < 3 - 5x \leq 10$$

$$(d) \frac{2x+7}{5} < \frac{5x-3}{2} \quad (e) \frac{1}{2} \leq \frac{5x-6}{4} < 7 \quad (f) 5x - x^2 < 6$$

$$(g) t^2 + (t+1)^2 > (t+2)^2 \quad (h) 5 > \frac{x+3}{x} \quad (i) \frac{-9x^2}{x^2-9} \geq 0$$

5. Find the domain of the following functions (or find the values of x for which the following do not represent real numbers):

$$(a) \sqrt{x^2-25} \quad (b) \sqrt{\frac{x-4}{x+4}} \quad (c) \frac{2x}{x^2-9}$$

$$(d) x^{2/3} - x^{1/3} - 6 \quad (e) \sqrt{x^2+1} \quad (f) \sqrt{(x+2)^2(3-x)(x)}$$

6. Solve the following:

$$(a) 3|5-2x|+4=9 \quad (b) 4|2x-7|+5<19 \quad (c) |5x^2-1|<-1$$

$$(d) |2x-5|=|8x+3| \quad (e) |2x-1|^2 \leq |x+3|^2$$

Solutions

1. (a) $\frac{x-1}{5x+3}$ (b) $\frac{(x-3)^2}{x(x+3)}$ (c) $\frac{1}{x-1}$
 (d) $\frac{3x^2-x^4}{(x^2+1)^3}$ (e) $\frac{1}{x^2-1}$ (f) $\frac{x^2-2x+1}{x}$
2. (a) $\frac{x\sqrt[3]{6xy^2z}}{3yz}$ (b) $\frac{x-\sqrt{x}}{x-1}$ (c) $\frac{x\sqrt{x-2}}{x-2}$ (d) $\frac{x+3\sqrt{x}+2}{x-1}$
3. (a) $x = 13/3$ (b) $x = -1$ (c) $x = \frac{1 \pm \sqrt{33}}{8}$ (d) $x = 27, x = 8$
 (e) candidates: $x = 0, x = 8$; winner: $x = 8$ (f) candidates: $x = 2, x = 7$; winner: $x = 7$
4. (a) $(32, \infty)$ (b) $(-1/2, 6]$ (c) $[-7/5, 3/5)$ (d) $(29/21, \infty)$
 (e) $[8/5, 34/5)$ (f) $(-\infty, 2) \cup (3, \infty)$ (g) $(-\infty, -1) \cup (3, \infty)$
 (h) $(-\infty, 0) \cup (3/4, \infty)$ (i) $(-3, 3) \cup \{0\}$
5. (a) $(-\infty, -5] \cup [5, \infty)$ (b) $(-\infty, -4) \cup [4, \infty)$ (c) $\mathbb{R} \setminus \{-3, 3\}$ (d) \mathbb{R}
 (e) \mathbb{R} (f) $[0, 3]$
6. (a) $x = 5/3, x = 10/3$ (b) $(7/4, 21/4)$ (c) No solution
 (d) $x = -4/3, x = 1/5$ (e) $(-\infty, -5 - \sqrt{7}] \cup [5 + \sqrt{7}, \infty)$